TA Session 1: Notes*

S.A.300.699: Accelerated Microeconomics

September 12, 2018

This set of notes is intended to shore up some select foundational *concepts*. The practice problems provide opportunity to practice the *methods* that use these concepts. These notes only cover a few areas of interest; lecture slides provide the entire set of important content.

1 Utility and Indifference Curves

Utility: A general measure of welfare. Not an actual number (no one says "I'm at happiness level 7 today!"), but we work with actual numbers to usefully represent relatively "higher" and "lower" utility.

Utility function: A mathematical expression that captures all the different possible levels of welfare given your relative preferences for two[1] goods: $U = U(x, y)$.

For example, what is your relative preference for econ textbooks compared to movie tickets? Say x = quantity of econ textbooks and y = quantity of movie tickets. Perhaps you value them both "somewhat" and equally:

$$U = x^{1/2}y^{1/2}.$$

Or perhaps you value them both "a lot" and equally:

$$U = x^1y^1.$$

What if you value econ textbooks "a TON" and movie tickets "not much at all"?

$$U = x^{3/2}y^{1/5}.$$

And if you like econ texbooks a lot and have literally no interest in movie tickets? Perhaps your utility function might end up with y having no weight at all (i.e. y has no effect on your happiness or welfare):

$$U = x^1.$$

Indifference curve: In addition to the textbook definition, here are a few useful interpretations to keep in mind:

1) The curve that shows me what combinations of quantities of goods ("bundles") would give me the exact same utility.

2) A solution for a utility function at a *specific value*, for example a curve representing when $U = 1$.

3) Building off of the last interpretation: a useful way to visually compare the bundles of goods that yiled different levels of utility. For example, by graphing indifference curves for $U = 1$, $U = 2$, and $U = 3$, I can see what I would need to do to achieve each level. Moving to the next step, I can add to this graph my *budget constraint* to visually find what level of utility I can afford (literally) to achieve.

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*This document produced using XƎLATEX. These notes are my own expressions of selected concepts and reflect conversations from the TA session. They are not comprehensive. Let me know of any mistakes you may find!

[1]Could we look at more than two goods? Sure. Easily? Eh... Are we going to? Nope.

2 Budgets

Budget constraint: An unfortunate reality for most of us (but fortunate for resource conservation).

Budget line: A line that represents all combinations of two goods that you could purchase (assuming you use all your money; if you don't you end up somewhere between the line and the origin at (0,0)). Put differently, the curve of your *consumption possibilities.* Also: easier to construct than commonly thought.

To build your budget constraint from the bottom up, do what comes intuitively: see how much you could buy of one good if you put all your money toward it; then buy one less of that good to buy some of the other one; repeat until you get to the other extreme of putting all your money into the other good. (Hint: it'll be a straight line!)

To build your budget constraint the fast way (*after* you have built your intuition for what this line shows us):

1) Write the following: $m = p_x x + p_y y$;

2) Set the following: $m =$ your total budget, $p_x =$ the price of the first good, $x =$ the quantity of the first good purchased, $p_y =$ the price of the second good, $y =$ the quantity of the second good purchased;

3) Graph your beautiful new construction (it helps to put it in "y =" form but you can do it intuitively once you've done it a few times).

You'll soon be looking at where this budget line touches (is tangent to) indifference curves, which show you how much utility you can get for a given budget... and you might guess where that is headed.

3 Derivatives

3.1 Regular derivatives

Conceptually: They capture *change.* They are also how we think at the *margins* (a favorite refrain of economists, particularly Mankiw).

Usefully: It can help us answer important quesitons such as: how much *extra utility* would I gain from a small *change in quantity x*? We call the answer the **marginal utility** of x: $MU_x = \frac{\partial d}{\partial dx} U(x, y)$. Similarly for y.

Refresher on regular derivatives: The function type that we typically care about and encounter in this course takes the following form (where a is a constant):

$$f(x) = x^a.$$

The derivative is:

$$\frac{d}{dx}(f(x)) = ax^{a-1}.$$

We encounter a lot of negative exponents. What do we do with $f(x) = x^{-1/2}$? Don't worry about things looking odd; they still work exactly the same way. It becomes: $\frac{d}{dx}(x^{-1/2}) = (-\frac{1}{2})x^{-3/2}$.

3.2 Partial derivatives

Conceptually: A way to took at change in *only the variable specified*; this means that any other variables are, during the derivation, treated exactly the same as *constants.*

Practically: Actually no harder than normal derivatives (in any cases that we will encounter).

"Real life" example: Let's say you have a start-up company building a magic formula that advises panicked students on what to do in the last few days before an exam in order to get an A. (I've heard of crazier ideas.) You do a lot of research and you find that the only two things that matter are *number of hours of study* and *number of hours of sleep.* These things are both *scarce* and *in competition* (ideas we'll discuss more later but which are fairly intuitive). But as you test your magic formula, you come across an important question: you see a combined effect, but how much does the outcome depend on *each one independently*?

To look at the change in outcome due to a small change in only one of the variables, intuitively, that means you hold the other one constant. In fancier language, you might say *ceteris paribus.* This is exactly what a partial derivative lets you look at. (And you could have as many variables as you can stomach and *still* use partial derivatives the same way to understand how each of them works!)

Marginal Utility (the key example): Say that $U(x, y) = x^{2/3}y^{3/4}$. You want to find the effect on U due to a small change in x. Recall that this is the *marginal utility of* x. It looks like this:

$$MU_x = \frac{\partial}{\partial x}U(x, y) = \frac{\partial}{\partial x}(x^{2/3}y^{3/4}).$$

What happens to y when we take this derivative? We are looking at x. Because of that, y acts like a constant! Thus, we simply get:

$$MU_x = \frac{2}{3}x^{-1/3}y^{3/4}.$$

If this seems difficult, imagine (only *imagine*) that $y = 16$, which would mean that $y^{3/4} = 8$. Then try again: what is the derivative, in terms of x, of $(x^{2/3} * 8)$? Nothing happens to the 8. You just get $\frac{2}{3}x^{-1/3} * 8$ (you would simplify that, but I'm separating it just to make clear that the 8 is still hanging out). In the same way, nothing happens to that $y^{3/4}$ when you take the partial derivative in terms of x.

4 Closing Comments

Why is this stuff necessary? It is, or will soon be, very useful. Not just for this class, but for future econ classes; and not just for classes, but for thinking about a host of important problems in the real world. How *should* our finite resources be allocated? To answer that, we must ask (for example): what would be the consequences of a small shift in the current allocation (marginal utility and partial derivatives)? Which goods have the greatest impact on people's welfare (utility functions and indifference curves? How can we match limited financing to the best outcomes possible (budget constraints - more on these next week)? These are the types of questions these concepts and methods help us begin to approach.